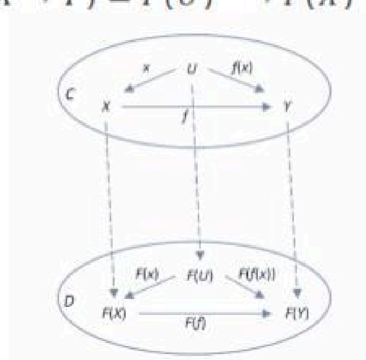
Categories for AI Bootcamp

Mastering the essence of Category theory for AI rigorously in 12 weeks!

$$F(U \xrightarrow{x} X \xrightarrow{f} Y) = F(U) \xrightarrow{F(x)} F(X) \xrightarrow{F(f)} F(Y)$$



Quantum Formalism CIC

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1. **INTRODUCTION**

Category theory is a fascinating branch of mathematics that has its roots back in the late forties in algebraic topology through the seminal works of Samuel Eilenberg and Saunders Mac Lane. It emerged as a powerful framework to unify and generalise various mathematical concepts and structures.

In the 1940s, algebraic topology was a rapidly developing field that sought to understand topological spaces and their properties through algebraic methods. Samuel Eilenberg and Saunders Mac Lane introduced category theory in their 1945 paper "General Theory of Natural Equivalences" to formalise the relationships between different topological spaces and continuous maps. They introduced the fundamental concepts of categories, functors, and natural transformations, which provided a new way to abstract and generalise mathematical structures.

The key basic concepts of Categories are:

- **Categories**: A category consists of objects and morphisms (arrows) between those objects, adhering to specific rules about composition and identity.
- **Functors**: Functors are mappings between categories that preserve the structure of categories, i.e., they map objects to objects and morphisms to morphisms in a way that respects composition and identities.
- **Natural Transformations**: Natural transformations provide a way of transforming one functor into another while respecting the structure of the categories involved.

If you have taken a linear algebra course, you are familiar with vector spaces. Vector spaces indeed form a category in category theory, denoted **Vect**. In this category, the objects are vector spaces, and the morphisms (arrows) between these objects are linear maps. Linear maps are functions that preserve the vector space structure, meaning they respect vector addition and scalar multiplication.

When dealing with finite-dimensional vector spaces, these linear maps can be represented by matrices. Hence, in the category **Vect**, composition of morphisms (linear maps) corresponds to matrix multiplication when dealing with finite-dimensional spaces. The identity morphism for each object (vector space) is represented by the identity matrix, which leaves any vector unchanged under multiplication.

This categorical perspective provides a unified framework to study and understand various properties and operations on vector spaces, offering deeper insights and powerful tools that extend beyond ordinary linear algebra into other fields such as functional analysis, and group theory.

2. COURSE MOTIVATION

Fast forward to the present, category theory has found applications in a wide range of fields outside mathematics, including computer science, database theory, functional programming, game theory, machine learning, quantum information science, and theoretical physics. Unfortunately, due to its abstract nature, category theory has remained somewhat inaccessible to industry professionals working on cutting-edge products that could significantly impact their industries. While popular online platforms such as Coursera offer self-paced and video-based courses such as the 'Mathematics for Machine Learning and Data Science Specialization,' they often avoid deeper mathematical subjects like category theory to cater to a broader audience.

Category theory offers a robust framework for thinking about abstractions and relationships, making it a valuable tool in artificial intelligence and computer science. Some key applications include;

1. **Compositionality**: In AI, compositionality refers to the ability to build complex systems from simpler components. Category theory provides the mathematical underpinning for understanding and implementing compositional systems. See the recorded online workshop 'The Challenge of Compositionality for AI' for more information.
2. **Functorial Data Modelling**: Functors can model data transformations and pipelines, ensuring that the structure and relationships within the data are preserved throughout various processing stages. See an example of this via these slides on 'Categorical databases'.
3. **Learning and Representation**: Categories and functors can model various aspects of machine learning, including neural network architectures and the relationships between different types of data representations. You will find a lot of useful papers and examples of this via this awesome GitHub repository by Bruno

Gavranović.

3. WHY TAKE THIS COURSE?

This course is designed for current and aspiring machine learning researchers and engineers who have a basic understanding of linear algebra and differential calculus. No prior knowledge of category theory is assumed. Each module of the course is self-contained, providing all the necessary background information as we progress through the material.

Additionally, unlike existing online mathematical courses for ML delivered via platforms like Coursera, this bootcamp offers unique features:

- **Live Lectures:** Attend live sessions with a PhD-level mathematician, focusing on core concepts from category theory that are relevant to the current ML trends.
- **Replays (exclusively available to participants):** If you miss a session, recordings will be available.
- **Live Tutorial Sessions:** Participate in live tutorials to learn how to prove theorems using the language of category theory and to construct core notions such as functors, and more. These sessions will not be recorded to encourage active participation and to create a safe environment where you can freely explore ideas and make mistakes without concern.
- **Feedback:** Receive feedback on assignments to help enhance your proof-writing and computational skills.
- **Applications in ML:** Learn how the category theory concepts covered in the lectures are applied in real-world scenarios through guest lectures by industry professionals and academic experts.
- **Community:** Join a wider network of ML professionals who value the role of mathematics in creating impactful solutions.
- **Career Fair (Optional):** Pitch yourself to select companies from ML to quantum computing looking for new talent.

This specialisation leverages pedagogical techniques, honed over four years of offering free mathematical courses to the QF community. Our approach is designed to help you learn quickly and intuitively, while simultaneously developing your abstract mathematical skills, equipping you to develop advanced ML algorithms with confidence.

4. **COURSE STRUCTURE**

The bootcamp runs from September 2024 to November 2024, requiring just 6 hours per week from our learners:

- 1 hour of live lectures
- 2 hours of live tutorials
- 3 hours for homework assignments (estimate)

REFERENCES

- ***Books on Category Theory***

1. *Mac Lane, S. (1998). Categories for the Working Mathematician. 2nd ed., New York: Springer-Verlag.*
2. *Spivak, D. I. (2014). Category Theory for the Sciences. Cambridge, MA: MIT Press.*
3. *Leinster, T. (2014). Basic Category Theory. Cambridge: Cambridge University Press.*

- ***Free Useful Online Resources***

1. *Categorical Deep Learning (by Bruno Gavranović et al).*
2. *Category Theory ∩ Machine Learning (Bruno Gavranović).*